HIGHER SECONDARY SECOND YEAR

MATHEMATICS

MODEL QUESTION PAPER – 1

Time Allowed: 15 Min + 3.00 Hours] **[Maximum Marks:90**

Instructions:
(a) Check the question paper for fairness of printing. If there is any lack of fairness, inform the Hall Supervisor immediately.
(b) Use **Blue** or **Black** ink to write and underline and pencil to draw diagrams.

PART – I

Note: (i) **All** questions are **compulsory.** $20\times1=20$

(ii) Choose the correct or most suitable answer from the given **four** alternatives. Write the option code and the corresponding answer.

1. If A and B are orthogonal, then $(AB)^T(AB)$ is

(a) A (b) B (c) I (d) A^T

2. If A is a non-singular matrix such that $A^{-1}=\begin{bmatrix}5 & 3\\ -2 & -1\end{bmatrix}$, then $(A^T)^{-1}=$

(a) $\begin{bmatrix}-5 & 3\\ 2 & 1\end{bmatrix}$ (b) $\begin{bmatrix}5 & 3\\ -2 & -1\end{bmatrix}$ (c) $\begin{bmatrix}-1 & -3\\ 2 & 5\end{bmatrix}$ (d) $\begin{bmatrix}5 & -2\\ 3 & -1\end{bmatrix}$

3. $i^n+i^{n+1}+i^{n+2}+i^{n+3}$ is

(a) 0 (b) 1 (c) -1 (d) i

4. The product of all four values of $\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)^{\frac{3}{4}}$ is

(a) -2 (b) -1 (c) 1 (d) 2

5. A polynomial equation in x of degree n always has

(a) n distinct roots (b) n real roots
(c) n imaginary roots (d) at most one root.

6. If $\sin^{-1}x+\sin^{-1}y=\frac{2\pi}{3}$, then $\cos^{-1}x+\cos^{-1}y$ is equal to

(a) $\frac{2\pi}{3}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{6}$ (d) π

7. The equation of the directrix of the parabola $y^2=x+4$ is

(a) $x=\frac{15}{4}$ (b) $x=-\frac{15}{4}$ (c) $x=-\frac{17}{4}$ (d) $x=\frac{17}{4}$

8. The circle $x^2 + y^2 = 4x + 8y + 5$ intersects the line $3x - 4y = m$ at two distinct points if

(a) $15 < m < 65$ (b) $35 < m < 85$ (c) $-85 < m < -35$ (d) $-35 < m < 15$

9. If the line $\frac{x-1}{3} = \frac{y-2}{4} = \frac{z-3}{\lambda}$ is perpendicular to the plane $\vec{r}\cdot\left(2\hat{i}+3\hat{j}+4\hat{k}\right)=0$, then the value of λ is

(a) $-\frac{13}{4}$ (b) -13 (c) -4 (d) $-\frac{1}{4}$

10. Distance from the origin to the plane $3x - 6y + 2z + 7 = 0$ is

(a) 0 (b) 1 (c) 2 (d) 3

11. The curve $y = ax^4 + bx^2$ with $ab > 0$

(a) has no horizontal tangent (b) is concave up

(c) is concave down (d) has no points of inflection

12. The function $f(x) = \sqrt[3]{4 - x^2}$ has a vertical tangent at

(a) $x = 0$ (b) $x = 2$ and $x = -2$

(c) $x = 0, x = 2$ and $x = -2$ (d) $x = 1$ and $x = -1$

13. If we measure the side of a cube to be 4 cm with an error of 0.1 cm, then the error in our calculation of the volume is

(a) 0.4 cu.cm (b) 0.45 cu.cm (c) 2 cu.cm (d) 0.8 cu.cm

14. $\int_0^{\infty} e^{-3x} x^2 \, dx =$

(a) $\frac{7}{27}$ (b) $\frac{5}{27}$ (c) $\frac{4}{27}$ (d) $\frac{2}{27}$

15. The value of $\int_0^{\pi} (\sin x + \cos x)\, dx$

(a) 1 (b) 2 (c) 0 (d) 4

16. P is the amount of certain substance left in after time t. If the rate of evaporation of the substance is proportional to the amount remaining, then (where $k > 0$)

(a) $P = ce^{kt}$ (b) $P = ce^{-kt}$ (c) $P = ckt$ (d) $Pt = c$

17. The solution of the differential equation $\frac{dy}{dx} = \frac{y}{x} + \frac{\phi\left(\frac{y}{x}\right)}{\phi'\left(\frac{y}{x}\right)}$ is

(a) $x\phi\left(\frac{y}{x}\right) = k$ (b) $\phi\left(\frac{y}{x}\right) = kx$ (c) $y\phi\left(\frac{y}{x}\right) = k$ (d) $\phi\left(\frac{y}{x}\right) = ky$

18. A rod of length $2l$ is broken into two pieces at random. The probability density function of the shorter of the two pieces is

$$f(x)=\begin{cases}\frac{1}{l} & 0<x<l \\ 0 & l\le x<2l\end{cases}$$

The mean and variance of the shorter of the two pieces are respectively

(a) $\frac{l}{2}, \frac{l^2}{3}$ (b) $\frac{l}{2}, \frac{l^2}{6}$ (c) $l, \frac{l^2}{12}$ (d) $\frac{l}{2}, \frac{l^2}{12}$

19. If X is a binomial random variable with expected value 6 and variance 2.4, Then $P\{X=5\}$ is

(a) $\binom{10}{5}\left(\frac{3}{5}\right)^6\left(\frac{2}{5}\right)^4$ (b) $\binom{10}{5}\left(\frac{3}{5}\right)^5$ (c) $\binom{10}{5}\left(\frac{3}{5}\right)^4\left(\frac{2}{5}\right)^6$ (d) $\binom{10}{5}\left(\frac{3}{5}\right)^5\left(\frac{2}{5}\right)^5$

20. The operation $*$ is defined by $a*b=\frac{ab}{7}$. It is it not a binary operation on

(a) $\mathbb{Q}^+$ (b) $\mathbb{Z}$ (c) $\mathbb{R}$ (d) $\mathbb{C}$

PART – II

Note: (i) Answer any **SEVEN** questions. $7\times2=14$

(ii) Question number **30 is compulsory.**

21. Prove that $\begin{bmatrix}\cos\theta & -\sin\theta \\ \sin\theta & \cos\theta\end{bmatrix}$ is orthogonal.

22. Find the modulus of $\frac{1-i}{3+i}+\frac{4i}{5}$.

23. Find the value of $\tan^{-1}\left(\tan\left(-\frac{\pi}{6}\right)\right)$

24. Find the centre and radius of the circle $3x^2+3y^2-12x+6y-9=0$.

25. Verify whether the line $\frac{x-3}{-4}=\frac{y-4}{-7}=\frac{z+3}{12}$ lies in the plane $5x-y+z=8$.

26. Evaluate : $\int_{-\log 2}^{\log 2} e^{-|x|}dx$

27. Determine the order and degree (if exists) of the differential equation

$$\left(\frac{d^2y}{dx^2}\right)^2+\left(\frac{dy}{dx}\right)^2=x\sin\left(\frac{d^2y}{dx^2}\right).$$

28. Suppose a discrete random variable can only take the values 0, 1, and 2. The probability mass function is defined by $f(x)=\begin{cases}\dfrac{x^2+1}{k}, & \text{for } x=0,1,2 \\ 0 & \text{otherwise}\end{cases}$. Find the value of k.

29. Verify the associative property under the binary operation $*$ defined by $a*b=a^b,\ \forall\ a,b\in\mathbb{N}$

30. Evaluate $\lim\limits_{x\to 0}\dfrac{xe^x-\sin x}{x}$

PART– III

Note: (i) Answer any **SEVEN** questions. $7\times3=21$

(ii) Question number **40 is compulsory.**

31. Find the rank of the matrix by row reduction method: $\begin{bmatrix}1 & 1 & 1 & 3\\ 2 & -1 & 3 & 4\\ 5 & -1 & 7 & 11\end{bmatrix}$

32. Show that the equation $z^3+2\bar{z}=0$ has five solutions.

33. Solve the equation $x^3-5x^2-4x+20=0$.

34. If the equation $3x^2+(3-p)xy+qy^2-2px=8pq$ represents a circle, find p and q. Also determine the centre and radius of the circle.

35. Find the points on the curve $y=x^3-6x^2+x+3$ where the normal is parallel to the line $x+y=1729$..

36. If $u(x,y)=\dfrac{x^2+y^2}{\sqrt{x+y}}$, prove that $x\dfrac{\partial u}{\partial x}+y\dfrac{\partial u}{\partial y}=\dfrac{3}{2}u$.

37. Evaluate : $\displaystyle\int_0^1\frac{2x}{1+x^2}dx$

38. Solve : $\cos x\cos y\,dy-\sin x\,\sin y\,dx=0$

39. If μ and σ^2 are the mean and variance of the discrete random variable X, and $E(X+3)=10$ and $E(X+3)^2=116$, find μ and σ^2.

40. If $\vec{a},\vec{b},\vec{c}$ are three vectors prove that $\left[\vec{a}+\vec{b},\ \vec{b}+\vec{c},\ \vec{c}+\vec{a}\right]=2\left[\vec{a},\vec{b},\vec{c}\right]$

PART – IV

Note: Answer **all** the questions. $7\times5=35$

41. (a) Examine the consistency of the system of equations $4x+3y+6z=25$, $x+5y+7z=13$, $2x+9y+z=1$. If it is consistent then solve.

(OR)

(b) If $z=x+iy$ and $\arg\left(\dfrac{z-i}{z+2}\right)=\dfrac{\pi}{4}$, then show that $x^2+y^2+3x-3y+2=0$.

42. (a) Solve the equation : $6x^4-35x^3+62x^2-35x+6=0$.

(OR)

(b) At a water fountain, water attains a maximum height of $4m$ at horizontal distance of $0.5\ m$ from its origin. The flow is from the origin and the path of water is a parabola open upwards, find the height of water at a horizontal distance of $0.75m$ from the point of origin.

43. (a) If $\vec{a}=\hat{i}-\hat{j}$, $\vec{b}=\hat{i}-\hat{j}-4\hat{k}$, $\vec{c}=3\hat{j}-\hat{k}$ and $\vec{d}=2\hat{i}+5\hat{j}+\hat{k}$, verify that $\left(\vec{a}\times\vec{b}\right)\times\left(\vec{c}\times\vec{d}\right)=\left[\vec{a},\vec{b},\vec{d}\right]\vec{c}-\left[\vec{a},\vec{b},\vec{c}\right]\vec{d}$

(OR)

(b) Find the vector and Cartesian equations of the plane $\vec{r}=\left(6\hat{i}-\hat{j}+\hat{k}\right)+s\left(-\hat{i}+2\hat{j}+\hat{k}\right)+t\left(-5\hat{i}-4\hat{j}-5\hat{k}\right)$.

44. (a) A hollow cone with base radius a cm and height b cm is placed on a table. Show that the volume of the largest cylinder that can be hidden underneath is $\frac{4}{9}$ times volume of the cone.

(OR)

(b) Expand $\log(1+x)$ as a Maclaurin's series upto 4 non-zero terms for $-1<x\leq 1$.

45. (a) If $u=xyz$, $x=e^{-t}, y=e^{-t}\sin t, z=\sin t$ find $\frac{du}{dt}$

(OR)

(b) Solve : $\left(y-e^{\sin^{-1}x}\right)\frac{dx}{dy}+\sqrt{1-x^2}=0$.

46. (a) A multiple choice examination has ten questions, each question has four distractors with exactly one correct answer. Suppose a student answers by guessing and if X denotes the number of correct answers, find (i) binomial distribution (ii) probability that the student will get seven correct answers (iii) at least one correct answer.

(OR)

(b) Show that $p\leftrightarrow q\equiv(p\wedge q)\vee(\neg p\wedge\neg q)$.

47. (a) Find the area enclosed by the curve $y=-x^2$ and the straight line $x+y+2=0$.

(OR)

(b) If $\cos^{-1}x+\cos^{-1}y+\cos^{-1}z=\pi$ and $0<x,y,z<1$, then show that $x^2+y^2+z^2+2xyz=1$.

HIGHER SECONDARY SECOND YEAR

MATHEMATICS

MODEL QUESTION PAPER – 2

Time Allowed: 15 Min + 3.00 Hours] **[Maximum Marks:90**

Instructions: (a) Check the question paper for fairness of printing. If there is any lack of fairness, inform the Hall Supervisor immediately.

(b) Use **Blue** or **Black** ink to write and underline and pencil to draw diagrams.

PART – I

Note: (i) **All** questions are **compulsory.** $20\times1=20$

1. If $A=\begin{bmatrix}7 & 3\\ 4 & 2\end{bmatrix}$, then $9I-A=$

(a) A^{-1} (b) $\frac{A^{-1}}{2}$ (c) $3A^{-1}$ (d) $2A^{-1}$

2. If $A=\begin{bmatrix}3 & -3 & 4\\ 2 & -3 & 4\\ 0 & -1 & 1\end{bmatrix}$, then adj(adj A) is

(a) $\begin{bmatrix}3 & -3 & 4\\ 2 & -3 & 4\\ 0 & -1 & 1\end{bmatrix}$ (b) $\begin{bmatrix}6 & -6 & 8\\ 4 & -6 & 8\\ 0 & -2 & 2\end{bmatrix}$ (c) $\begin{bmatrix}-3 & 3 & -4\\ -2 & 3 & -4\\ 0 & 1 & -1\end{bmatrix}$ (d) $\begin{bmatrix}3 & -3 & 4\\ 0 & -1 & 1\\ 2 & -3 & 4\end{bmatrix}$

3. The area of the triangle formed by the complex numbers z, iz and $z+iz$ in the Argand's diagram is

(a) $\frac{1}{2}|z|^2$ (b) $|z|^2$ (c) $\frac{3}{2}|z|^2$ (d) $2|z|^2$

4. All complex numbers z which satisfy the equation $\left|\frac{z-6i}{z+6i}\right|=1$ lie on the

(a) real axis (b) imginary axis (c) circle (d) ellipse

5. If $\cot^{-1}2$ and $\cot^{-1}3$ are two angles of a triangle, then the third angle is

(a) $\frac{\pi}{4}$ (b) $\frac{3\pi}{4}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{3}$

6. The range of $\sec^{-1}x$ is

(a) $[0,\pi]\setminus\left\{\frac{\pi}{2}\right\}$ (b) $[0,\pi]$ (c) $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ (d) $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

7. $P(x, y)$ be any point on $16x^2 + 25y^2 = 400$ with foci $F_1(3, 0)$ and $F_2(-3, 0)$ then $PF_1 + PF_2$ is

(a) 8 (b) 6 (c) 10 (d) 12

8. If the two tangents drawn from a point P to the parabola $y^2 = 4x$ are at right angles then the locus of P is

(a) $2x + 1 = 0$ (b) $x = -1$ (c) $2x - 1 = 0$ (d) $x = 1$

9. The locus of the point whose distance from $(-2,0)$ is $\frac{2}{3}$ times its distance from the line $x = -\frac{9}{2}$ is

(a) a parabola (b) a hyperbola (c) an ellipse (d) a circle

10. The angle between the line $\vec{r} = (\hat{i} + 2\hat{j} - 3\hat{k}) + t(2\hat{i} + \hat{j} - 2\hat{k})$ and the plane $\vec{r} \cdot (\hat{i} + \hat{j}) + 4 = 0$ is

(a) 0° (b) 30° (c) 45° (d) 90°

11. If the rate of increase of the radius of a circle is 5 cm/sec, then the rate of increase of its area when the radius is 20 cm, will be

(a) 10π (b) 20π (c) 200π (d) 400π

12. .Angle between $y^2 = x$ and $x^2 = y$ at the origin is

(a) $\tan^{-1}\frac{3}{4}$ (b) $\tan^{-1}\left(\frac{4}{3}\right)$ (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{4}$

13. If $w(x,y) = x^y, x > 0$, then $\frac{\partial w}{\partial x}$ is equal to

(a) $x^y \log x$ (b) $y \log x$ (c) yx^{y-1} (d) $x \log y$

14. $\int_0^a \frac{1}{4+x^2} dx = \frac{\pi}{8}$ then a is

(a) 4 (b) 1 (c) 3 (d) 2

15. The area bounded by the curve $y^2 = 4x$ and the lines $x = 1, x = 4$ and x – axis in the first quadrant is

(a) $\frac{11}{3}$ (b) $\frac{17}{3}$ (c) $\frac{28}{3}$ (d) $\frac{31}{3}$

16. If the solution of the differential equation $\frac{dy}{dx} = \frac{ax+3}{2y+f}$ represents a circle, then the value of a is

(a) 2 (b) −2 (c) 1 (d) −1

17. A random variable X has binomial distribution with $n = 25$ and $p = 0.8$ then standard deviation of X is

(a) 6 (b) 4 (c) 3 (d) 2

18. Suppose that X takes on one of the values 0, 1, and 2. If for some constant c, $P(X = i) = k P(X = i-1)$ for $i = 1,2$ and $P(X = 0) = \frac{1}{7}$. Then the value of k is

(a) 1 (b) 2 (c) 3 (d) 4

19. Which one is the inverse of the statement $(p \vee q) \rightarrow (p \wedge q)$?

(a) $(p \wedge q) \rightarrow (p \vee q)$
(b) $\neg(p \vee q) \rightarrow (p \wedge q)$
(c) $(\neg p \vee \neg q) \rightarrow (\neg p \wedge \neg q)$
(d) $(\neg p \wedge \neg q) \rightarrow (\neg p \vee \neg q)$

20. Which one of the following statements has the truth value T?

(a) $\sin x$ is an even function.

(b) Every square matrix is non-singular

(c) The product of complex number and its conjugate is purely imaginary

(d) $\sqrt{5}$ is an irrational number

PART – II

Note: (i) Answer any **SEVEN** questions. $7 \times 2 = 14$

(ii) Question number **30 is compulsory.**

21. Find z^{-1}, if $z = (2+3i)(1-i)$

22. Find the square root of $-6+8i$

23. Find the principal value of $\text{cosec}^{-1}(-\sqrt{2})$

24. Find the equation of the parabola whose end points of the latus rectum are $(4,-8)$ and $(4,8)$, centre is (0,0) and open rightward.

25. A particle is fired straight up from the ground to reach a height of s feet in t seconds, when $s = 128t - 16t^2$ .Compute the maximum height of the particle reached ?

26. If $f(x,y) = x^3 - 3x^2 + y^2 + 5x + 6$, then find f_x at $(1,-2)$

27. Find the differential equation of the curve represented by $xy = ae^x + be^{-x} + x^2$.

28. Establish the equivalence property: $p \rightarrow q \equiv \neg p \vee q$

29. Let $*$ be defined on $\mathbb{R}$ by $a * b = a + b + ab - 7$. Is $*$ binary on $\mathbb{R}$.

30. Evaluate : $\int_0^{\frac{\pi}{2}} \frac{\sin x}{1+\cos^2 x} dx$

PART– III

Note: (i) Answer any **SEVEN** questions. $7 \times 3 = 21$

(ii) Question number **40 is compulsory.**

31. Find the value of $\sum_{k=1}^{8}\left(\cos\frac{2k\pi}{9} + i\sin\frac{2k\pi}{9}\right)$.

32. Show that the equation $x^9 - 5x^5 + 4x^4 + 2x^2 + 1 = 0$ has atleast 6 imaginary solutions.

33. Prove that the length of the latus rectum of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is $\frac{2b^2}{a}$.

34. Find the altitude of a parallelepiped determined by the vectors $\vec{a} = -2\hat{i} + 5\hat{j} + 3\hat{k}$, $\vec{b} = \hat{i} + 3\hat{j} - 2\hat{k}$ and $\vec{c} = -3\hat{i} + \hat{j} + 4\hat{k}$ if the base is taken as the parallelogram determined by $\vec{b}$ and $\vec{c}$.

35. Examine the concavity for the function $f(x) = x^4 - 4x^3$.

36. Show that the value in the conclusion of the mean value theorem for $f(x) = Ax^2 + Bx + C$ on any interval $[a,b]$ is $\frac{a+b}{2}$.

37. Evaluate : $\int_0^{\frac{\pi}{2}} (\sin^2 x + \cos^4 x)\, dx$.

38. Assume that a spherical rain drop evaporates at a rate proportional to its surface area. Form a differential equation involving the rate of change of the radius of the rain drop.

39. The probability distribution of a random variable is given below

$X = x$	0	1	2	3	4	5	6	7
$P(X = x)$	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2 + k$

Then find $P(0 < X < 4)$.

40. If $A = \begin{bmatrix} 3 & -2 \\ \lambda & -2 \end{bmatrix}$, find the value of λ so that $A^2 = \lambda A - 2I$.

PART – IV

Note: Answer **all** the questions. $7 \times 5 = 35$

41. (a) Investigate for what values of λ and μ the system of linear equations $x + 2y + z = 7$, $x + y + \lambda z = \mu$, $x + 3y - 5z = 5$ has (i) no solution (ii) a unique solution

(OR)

(b) Find the sum of squares of the roots of the equation $2x^4 - 8x^3 + 6x^2 - 3 = 0$.

42. (a) Evaluate : $\sin\left[\sin^{-1}\left(\frac{3}{5}\right) + \sec^{-1}\left(\frac{5}{4}\right)\right]$

(OR)

(b) Find the foci and vertices of the hyperbola $4x^2 - 24x - 25y^2 + 250y - 489 = 0$.

43. (a) Find the vector and cartesian equations of the plane passing through the point $(1, -2, 4)$ and perpendicular to the plane $x + 2y - 3z = 11$ and parallel to the line $\frac{x+7}{3} = \frac{y+3}{-1} = \frac{z}{1}$.

(OR)

(b) Find the foot of the perpendicular drawn from the point $(5,4,2)$ to the line $\frac{x+1}{2}=\frac{y-3}{3}=\frac{z-1}{-1}$. Also, find the equation of the perpendicular.

44. (a) Find intervals of concavity and points of inflexion for the function

$$f(x)=\frac{1}{2}(e^{x}-e^{-x})$$

(OR)

(b) Evaluate : $\int_{0}^{2a} x^2\sqrt{2ax-x^2}\,dx$

45. (a) Find the area of the region bounded between the parabola $x^2=y$ and the curve $y=|x|$.

(OR)

(b) The equation of electromotive force for an electric circuit containing resistance and self-inductance is $E=Ri+L\frac{di}{dt}$, where E is the electromotive force given to the circuit, R the resistance and L, the coefficient of induction. Find the current i at time t when $E=0$.

46. (a) The probability density function of X is given by $f(x)=\begin{cases} ke^{-\frac{x}{3}} & \text{for } x>0 \\ 0 & \text{for } x\le 0 \end{cases}$

Find (i) the value of k (ii) $P(X<3)$.

(OR)

(b) Verify whether the compound proposition $(p\to q)\leftrightarrow(\neg p\to q)$ is a tautology or contradiction or contingency

47. (a) Solve $z^4=1-\sqrt{3}i$

(OR)

(b) If $f(x,y)=\log\sqrt{x^2+y^2}$, show that $\frac{\partial^2 f}{\partial x^2}+\frac{\partial^2 f}{\partial y^2}=0$

HIGHER SECONDARY SECOND YEAR

MATHEMATICS

MODEL QUESTION PAPER – 3

Time Allowed: 15 Min + 3.00 Hours] **[Maximum Marks:90**

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PART – I

Note: (i) **All** questions are **compulsory.** $20\times1=20$

(ii) Choose the correct or most suitable answer from the given **four** alternatives. Write the option code and the corresponding answer.

1. If $A=\begin{bmatrix} 1 & \tan\frac{\theta}{2} \\ -\tan\frac{\theta}{2} & 1 \end{bmatrix}$ and $AB=I$, then $B=$

(a) $\left(\cos^2\frac{\theta}{2}\right)A$ (b) $\left(\cos^2\frac{\theta}{2}\right)A^T$ (c) $(\cos^2\theta)I$ (d) $\left(\sin^2\frac{\theta}{2}\right)A$

2. If $x^a y^b = e^m, x^c y^d = e^n, \Delta_1 = \begin{vmatrix} m & b \\ n & d \end{vmatrix}, \Delta_2 = \begin{vmatrix} a & m \\ c & n \end{vmatrix}, \Delta_3 = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$, then the values of x and y are respectively,

(a) $e^{(\Delta_2/\Delta_1)}, e^{(\Delta_3/\Delta_1)}$ (b) $\log(\Delta_1/\Delta_3), \log(\Delta_2/\Delta_3)$

(c) $\log(\Delta_2/\Delta_1), \log(\Delta_3/\Delta_1)$ (d)) $e^{(\Delta_1/\Delta_3)}, e^{(\Delta_2/\Delta_3)}$

3. If $z=x+iy$ is a complex number such that $|z+2|=|z-2|$ then the locus of z is

(a) real axis (b) imaginary axis (c) ellipse (d) circle

4. The principal argument of the complex number $\frac{\left(1+i\sqrt{3}\right)^2}{4i\left(1-i\sqrt{3}\right)}$ is

(a) $\frac{2\pi}{3}$ (b) $\frac{\pi}{6}$ (c) $\frac{5\pi}{6}$ (d) $\frac{\pi}{2}$

5. The polynomial equation $x^3+2x+3=0$ has

(a) one negative and two real roots (b) one positive and two imaginary roots
(c) three real roots (d) no solution

6. $\tan^{-1}\left(\frac{1}{4}\right)+\tan^{-1}\left(\frac{2}{9}\right)$ is equal to

(a) $\frac{1}{2}\cos^{-1}\left(\frac{3}{5}\right)$ (b) $\frac{1}{2}\sin^{-1}\left(\frac{3}{5}\right)$ (c) $\frac{1}{2}\tan^{-1}\left(\frac{3}{5}\right)$ (d) $\tan^{-1}\left(\frac{1}{2}\right)$

7. The vertex of the parabola $x^2=8y-1$ is

(a) $\left(-\frac{1}{8},0\right)$ (b) $\left(\frac{1}{8},0\right)$ (c) $\left(-6,\frac{9}{2}\right)$ (d) $\left(\frac{9}{2},-6\right)$

8. Area of the greatest rectangle inscribed in the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is

(a) $2ab$ (b) ab (c) $\sqrt{ab}$ (d) $\frac{a}{b}$

9. If $\vec{a}=\hat{i}+\hat{j}+\hat{k}$, $\vec{b}=\hat{i}+\hat{j}$, $\vec{c}=\hat{i}$ and $(\vec{a}\times\vec{b})\times\vec{c}=\lambda\vec{a}+\mu\vec{b}$, then the value of $\lambda+\mu$ is

(a) 0 (b) 1 (c) 6 (d) 3

10. If the distance of the point (1,1,1) from the origin is half of its distance from the plane $x+y+z=k$, then the values of k are

(a) ±3 (b) ±6 (c) − 3, 9 (d) 3, −9

11. If $x+y=k$ is a normal to the parabola $y^2=16x$, then the value of k is

(a) 3 (b) 6 (c) 12 (d) 15

12. The number given by the Mean value theorem for the function $\frac{1}{x}, x\in[1,9]$ is

(a) 2 (b) 2.5 (c) 3 (d) 3.5

13. If $f(x)=\frac{x-1}{x+1}$, then its differential is given by

(a) $\frac{2}{(x+1)^2}dx$ (b) $-\frac{2}{(x+1)^2}dx$ (c) $\frac{x}{(x+1)^2}dx$ (d) $\frac{-x}{(x+1)^2}dx$

14. If $\int_0^x f(t)\,dt=x+\int_x^1 tf(t)\,dt$, then the value of $f(1)$ is

(a) $\frac{1}{2}$ (b) 2 (c) 1 (d) $\frac{3}{4}$

15. The value of $\int_0^1 \log\left(\frac{x}{1-x}\right)dx$

(a) 0 (b) 2 (c) 4 (d) 5

16. The degree of the differential equation $y(x)=1+\frac{dy}{dx}+\frac{1}{1\cdot 2}\left(\frac{dy}{dx}\right)^2+\frac{1}{1\cdot 2\cdot 3}\left(\frac{dy}{dx}\right)^3+....$ is

(a) 2 (b) 3 (c) 1 (d) 4

17. The population P in any year t is such that the rate of increase in the population is proportional to the population. Then $(k>0)$

(a) $P=ce^{kt}$ (b) $P=ce^{-kt}$ (c) $P=ckt$ (d) $P=c$

18. If the mean of a binomial distribution is 5 and its variance is 4, then the value of n and p are

(a) $\left(\frac{1}{5},25\right)$ (b) $\left(25,\frac{1}{5}\right)$ (c) $\left(25,\frac{4}{5}\right)$ (d) $\left(\frac{4}{5},25\right)$

19. The probability function of a random variable is defined as:

x	-2	-1	0	1	2
$f(x)$	k	$2k$	$3k$	$4k$	$5k$

Then E(X) is equal to:

(a) $\frac{1}{15}$ (b) $\frac{1}{10}$ (c) $\frac{1}{3}$ (d) $\frac{2}{3}$

20. Which one is the inverse of the statement $(p\vee q)\rightarrow(p\wedge q)$?

(a) $(p\wedge q)\rightarrow(p\vee q)$ (b) $\neg(p\vee q)\rightarrow(p\wedge q)$

(c) $(\neg p\vee\neg q)\rightarrow(\neg p\wedge\neg q)$ (d) $(\neg p\wedge\neg q)\rightarrow(\neg p\vee\neg q)$

PART – II

Note: (i) Answer any **SEVEN** questions. $7\times 2=14$

(ii) Question number **30 is compulsory.**

21. Show that $\left(2+i\sqrt{3}\right)^{10}+\left(2-i\sqrt{3}\right)^{10}$ is real

22. Find the value of $\sin^{-1}\left(\sin\left(\frac{5\pi}{4}\right)\right)$

23. Obtain the equation of the circle for which $(3,4)$ and $(2,-7)$ are the ends of a diameter.

24. Find the intercepts cut off by the plane $\vec{r}\cdot\left(6\hat{i}+4\hat{j}-3\hat{k}\right)=12$ on the coordinate axes.

25. For the function $f(x)=x^4-2x^2$, find all the values of c in $(-2,2)$ such that $f'(c)=0$

26. Evaluate : $\int_0^{\frac{\pi}{2}}\sin^{10}x\,dx$.

27 Show that the differential equation for the function $y=e^{-x}+mx+n$, where m and n are arbitrary constants is $e^{x}\left(\frac{d^2y}{dx^2}\right)-1=0$.

28. Find the mean of the distribution $f(x)=\begin{cases}3e^{-3x}, & 0<x<\infty \\ 0, & elsewhere\end{cases}$

29. On $\mathbb{Z}$, define $\otimes$ by $(m\otimes n)=m^n+n^m:\forall m,n\in\mathbb{Z}$. Is $\otimes$ binary on $\mathbb{Z}$?

30. If $A=\begin{bmatrix}2 & 3\\ 1 & 2\end{bmatrix}, B=\begin{bmatrix}4 & 0\\ 2 & 5\end{bmatrix}$, find $adj(AB)$.

PART– III

Note: (i) Answer any **SEVEN** questions. $7\times3=21$

(ii) Question number **40 is compulsory.**

31. Solve the following system of linear equations by matrix inversion method

$2x-y=8; 3x+2y=-2$

32. Find the value of $\dfrac{\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}}{\cos\frac{\pi}{3}-i\sin\frac{\pi}{3}}$.

33. Find the equation of the hyperbola with foci $(\pm3,5)$ and eccentricity $e=2$.

34. Find the cartesian equation of the plane passing through the line of intersection of the planes $\vec{r}\cdot(2\hat{i}-7\hat{j}+4\hat{k})=3$ and $3x-5y+4z+11=0$, and the point $(-2,1,3)$.

35. Prove that the function $f(x)=x-\sin x$ is increasing but not strictly on the real line. Also discuss for the existence of local extrema.

36. If $U=\log(x^3+y^3+z^3)$, find $\frac{\partial U}{\partial x}+\frac{\partial U}{\partial y}+\frac{\partial U}{\partial z}$.

37. Evaluate : $\int_0^{\pi} x^2\cos nx\,dx$, where n is a positive integer.

38. Solve : $\frac{dy}{dx}=e^{x+y}+x^2e^{x^3+y}$

39. The mean and variance of a binomial variate X are respectively 2 and 1.5. Find $P(X=0)$.

40. Find the magnitude and direction cosines of the moment about the point $(0,-2,3)$ of a force $\hat{i}+\hat{j}+\hat{k}$ whose line of action passes through the origin.

PART – IV

Note: Answer **all** the questions. $7\times5=35$

41. (a) If ax^2+bx+c is divided by $x+3, x-5$ and $x-1$, the remainders are $21, 61$ and 9 respectively. Find a, b and c. (OR)

(b) Simplify : $\left(-\sqrt{3}+3i\right)^{31}$

42. (a) Solve the equation $x^4-10x^3+26x^2-10x+1=0$. (OR)

(b) Solve for x : $\tan^{-1}x+2\cot^{-1}x=\frac{2\pi}{3}$

43. (a) For the ellipse $4x^2+y^2+24x-2y+21=0$, find the centre, vertices, foci and the length of latus rectum. (OR)

(b) By vector method, prove that $\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$.

44 (a) Find the absolute extrema of the function $f(x)=3x^4-4x^3$ on the interval $[-1,2]$.

(OR)

(b) For the function $f(x,y)=\frac{3x}{y+\sin x}$, find f_x, f_y, and show that $f_{xy}=f_{yx}$.

45 (a) Using integration find the area of the region bounded by triangle ABC, whose vertices A, B, and C are $(-1,1)$, $(3, 2)$, and $(0,5)$ respectively.

(OR)

(b) Solve : $y^2-x^2\frac{dy}{dx}=xy\frac{dy}{dx}$

46. (a) The cumulative distribution function of a discrete random variable is given by

$$F(x)=\begin{cases}0 & -\infty<x<-1\\ 0.15 & -1\le x<0\\ 0.35 & 0\le x<1\\ 0.60 & 1\le x<2\\ 0.85 & 2\le x<3\\ 1 & 3\le x<\infty\end{cases}$$

Find (*i*) the probability mass function (*ii*) $P(X<1)$ and (*iii*) $P(X\ge 2)$

(OR)

(b) Using truth table check whether the statements $\neg(p\vee q)\vee(\neg p\wedge q)$ and $\neg p$ are logically equivalent.

47. (a) Find the shortest distance between the straight lines $\frac{x-6}{1}=\frac{2-y}{2}=\frac{z-2}{2}$ and $\frac{x+4}{3}=\frac{y}{-2}=\frac{1-z}{2}$. (OR)

(b) Prove that among all the rectangles of the given area square has the least perimeter.

HIGHER SECONDARY SECOND YEAR

MATHEMATICS

MODEL QUESTION PAPER – 4

Time Allowed: 15 Min + 3.00 Hours] **[Maximum Marks:90**

Instructions: (a) Check the question paper for fairness of printing. If there is any lack of fairness, inform the Hall Supervisor immediately.

(b) Use **Blue** or **Black** ink to write and underline and pencil to draw diagrams.

PART – I

Note: (i) **All** questions are **compulsory.** $20\times1=20$

(ii) Choose the correct or most suitable answer from the given **four** alternatives. Write the option code and the corresponding answer.

1. The adjoint of 3×3 matrix P is $\begin{bmatrix}-1 & 2 & 2\\ 1 & 1 & 2\\ 2 & 2 & 1\end{bmatrix}$, then the possible value(s) of the determinant P is (are)

   (a) 3 (b) -3 (c) ±3 (d) $\pm\sqrt{3}$

2. If $x=\dfrac{-1+i\sqrt{3}}{2}$ then the value of x^2+x+1

   (a) 2 (b) $\dfrac{1}{2}$ (c) 0 (d) 1

3. The value of $\left(\dfrac{1+\sqrt{3}i}{1-\sqrt{3}i}\right)^{10}$ is

   (a) $cis\dfrac{2\pi}{3}$ (b) $cis\dfrac{4\pi}{3}$ (c) $-cis\dfrac{2\pi}{3}$ (d) $-cis\dfrac{4\pi}{3}$

4. A polynomial equation in x of degree n always has

   (a) n distinct roots (b) n real roots (c) n imaginary roots (d) atmost one root

5. $\sin^{-1}(\cos x)=\dfrac{\pi}{2}-x$ is valid for

   (a) $-\pi\le x\le 0$ (b) $0\le x\le\pi$ (c) $-\dfrac{\pi}{2}\le x\le\dfrac{\pi}{2}$ (d) $-\dfrac{\pi}{4}\le x\le\dfrac{3\pi}{4}$

6. If $\cot^{-1}\left(\sqrt{\sin\alpha}\right)+\tan^{-1}\left(\sqrt{\sin\alpha}\right)=u$, then $\cos 2u$ is equal to

   (a) $\tan^2\alpha$ (b) 0 (c) -1 (d) $\tan 2\alpha$

7. The locus of a point whose distance from $(-2,0)$ is $\frac{2}{3}$ times its distance from the line $x=-\frac{9}{2}$ is

(a) a parabola (b) a hyperbola (c) an ellipse (d) a circle

8. If $P(x,y)$ be any point on $4x^2+9y^2=36$, then the sum of the distances of P from the points $(\sqrt{5},0)$ and $(-\sqrt{5},0)$ is

(a) 4 (b) 8 (c) 6 (d) 18

9. If the plane $x+\alpha y+z-8=0$ has equal intercepts on the coordinate axes, the value of α is

(a) 1 (b) 2 (c) 8 (d) $\frac{1}{8}$

10. If the planes $\vec{r}\cdot(2\hat{i}-\lambda\hat{j}+\hat{k})=3$ and $\vec{r}\cdot(4\hat{i}+\hat{j}-\mu\hat{k})=5$ are parallel, then the value of λ and μ are

(a) $\frac{1}{2},-2$ (b) $-\frac{1}{2},2$ (c) $-\frac{1}{2},-2$ (d) $\frac{1}{2},2$

11. The position of a particle moving along a horizontal line of any time t is given by $s(t)=3t^2-2t-8$. For what values of t the particle is not moving?

(a) 0 (b) $\frac{1}{3}$ (c) 1 (d) 3

12. The minimum value of the function $|3-x|+9$ is

(a) 0 (b) 3 (c) 6 (d) 9

13. The abscissa of the point on the curve $f(x)=\sqrt{8-2x}$ at which the slope of the tangent is -0.25 ?

(a) -8 (b) -4 (c) -2 (d) 0

14. If $f(x)=\frac{x}{x+1}$, then its differential is given by

(a) $\frac{-1}{(x+1)^2}dx$ (b) $\frac{1}{(x+1)^2}dx$ (c) $\frac{1}{x+1}dx$ (d) $\frac{-1}{x+1}dx$

15. The solution of the differential equation $\frac{dy}{dx}+\frac{1}{\sqrt{1-x^2}}=0$ is

(a) $y+\sin^{-1}x=c$ (b) $x+\sin^{-1}y=0$ (c) $y^2+2\sin^{-1}x=c$ (d) $x^2+2\sin^{-1}y=0$

16. The differential equation of the family of curves $y=Ae^x+Be^{-x}$, where A and B are arbitrary constants is

(a) $\frac{d^2y}{dx^2}+y=0$ (b) $\frac{d^2y}{dx^2}-y=0$ (c) $\frac{dy}{dx}+y=0$ (d) $\frac{dy}{dx}-y=0$

17. The solution of the differential equation $\frac{dy}{dx}=e^x+2$ is

(a) $y=e^x+C$ (b) $y=2x+e^x+C$ (c) $y=2xe^x+C$ (d) $y=e^x+2Cx$

18. The random variable X has the probability density function

$f(x)=\begin{cases} ax+b & 0<x<1 \\ 0 & \text{otherwise} \end{cases}$ and $E(X)=\frac{7}{12}$, then a and b are respectively

(a) 1 and $\frac{1}{2}$ (b) $\frac{1}{2}$ and 1 (c) 2 and 1 (d) 1 and 2

19. In the set $\mathbb{R}$ of real numbers '$*$' is defined as follows. Which one of the following is not a binary operation on $\mathbb{R}$?

(a) $a*b=\min(a,b)$ (b) $a*b=\max(a,b)$

(c) $a*b=a$ (d) $a*b=a^b$

20. If $a*b=\sqrt{a^2+b^2}$ on the real numbers then $*$ is

(a) commutative but not associative (b) associative but not commutative

(c) both commutative and associative (d) neither commutative nor associative

PART – II

Note: (i) Answer any **SEVEN** questions. $7\times2=14$

(ii) Question number **30 is compulsory.**

21. If A is a non-singular matrix of odd order, prove that $|\text{adj}(A)|$ is positive.

22. Write the principal value of $\tan^{-1}\left[\sin\left(-\frac{\pi}{2}\right)\right]$

23. Identify the type of the conic $y^2+4x+3y+4=0$.

24. Find the foci of $9x^2-16y^2=144$.

25. Find the angle between the lines $2x=3y=-z$ and $6x=-y=-4z$.

26. Find the intervals of monotonicity for the function $f(x)=x^2-4x+4$.

27. Evaluate : $\int_0^1 \frac{\left(\sin^{-1}x\right)^2}{\sqrt{1-x^2}}dx$

28. Find the order and degree (if exists) of the differential equation $y\left(\frac{dy}{dx}\right)=\frac{x}{\left(\frac{dy}{dx}\right)+\left(\frac{dy}{dx}\right)^3}$.

29. If X is the random variable with distribution function $F(x)$ given by,

$F(x)=\begin{cases} 0, & x<0 \\ \frac{1}{2}\left(x^2+x\right) & 0\le x<1 \\ 1, & x\ge 1 \end{cases}$ then find the probability density function $f(x)$

30. If $\omega\neq1$ is a cubic root of unity and $(1+\omega)^7=A+B\omega$, then find A and B.

PART– III

$7\times3=21$

Note: (i) Answer any **SEVEN** questions.

(ii) Question number **40 is compulsory.**

31. Find the rank of the matrix $\begin{bmatrix} 4 & -2 & 6 & 8 \\ 1 & 1 & -3 & -1 \\ 15 & -3 & 9 & 21 \end{bmatrix}$.

32. If α and β are the roots of the quadratic equation $2x^2-7x+13=0$, construct a quadratic equation whose roots are α^2 and β^2.

33. The line $3x+4y-12=0$ meets the coordinate axes at A and B. Find the equation of the circle drawn on AB as diameter

34. Find the magnitude and direction cosines of the torque of a force represented by $3\hat{i}+4\hat{j}-5\hat{k}$ about the point with position vector $2\hat{i}-3\hat{j}+4\hat{k}$ acting through a point whose position vector is $4\hat{i}+2\hat{j}-3\hat{k}$.

35. Find the local extrema for the function $f(x)=x^2e^{-2x}$ using second derivative test.

36. A circular plate expands uniformly under the influence of heat. If it's radius increases from 10.5 cm to 10.75 cm, then find an approximate change in the area and the approximate percentage change in the area.

37. If $u=e^{2(x-y)}$, prove that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=u\log u$.

38. Evaluate $\int_0^{2\pi} x\log\left(\frac{3+\cos x}{3-\cos x}\right)dx$ using properties of integration.

39. Show that (i) $p\vee(\neg p)$ is a tautology (ii) $p\wedge(\neg p)$ is a contradiction.

40. The population of a city grows at the rate of 5 % per year. Calculate the time taken for the population doubles. [Given $\log 2=0.6912$]

PART – IV

$7\times5=35$

Note: Answer **all** the questions.

41. (a) Determine the values of λ for which the following system of equations $x+y+3z=0; 4x+3y+\lambda z=0; 2x+y+2z=0$ has

(i) a unique solution (ii) a non-trivial solution.

(OR)

(b) If $\cos\alpha+\cos\beta+\cos\gamma=\sin\alpha+\sin\beta+\sin\gamma=0$ then show that

(i) $\cos3\alpha+\cos3\beta+\cos3\gamma=3\cos(\alpha+\beta+\gamma)$ and

(ii) $\sin3\alpha+\sin3\beta+\sin3\gamma=3\sin(\alpha+\beta+\gamma)$

42. (a) Solve the equation $z^3+8i=0$, where $z\in\mathbb{C}$. (OR)

(b) Draw the curve $\sin x$ in the domain $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ and $\sin^{-1}x$ in $[-1,1]$.

43. (a) The eccentricity of an ellipse with its centre at the origin is $\frac{1}{2}$. If one of the directrix is $x=4$, then find the equation of the ellipse.

(OR)

(b) If the straight lines $\frac{x-1}{2}=\frac{y+1}{\lambda}=\frac{z}{2}$ and $\frac{x+1}{5}=\frac{y+1}{2}=\frac{z}{\lambda}$ are coplanar,find λ and cartesian equations of the planes containing these two lines.

44. (a) Find the angle between $y=x^2$ and $y=(x-3)^2$.

(OR)

(b) Let $w(x,y,z)=\frac{1}{\sqrt{x^2+y^2+z^2}},(x,y,z)\neq(0,0,0)$. Show that $\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}=0$.

45. (a) Using integration , find the area of the region bounded by the triangle whose vertices are $(-1,2),(1,5)$ and $(3,4)$.

(OR)

(b) Solve : $\left(1+2e^{x/y}\right)dx+2e^{x/y}\left(1-\frac{x}{y}\right)dy=0$.

46. (a) The probability density function of X is given by $f(x)=\begin{cases}16xe^{-4x} & \text{for } x>0\\ 0 & \text{for } x\leq 0\end{cases}$ Find the mean and variance of X.

(OR)

(b) Verify (i) closure property (ii) associative property (iii) existence of identity (iv) existence of inverse and (v) commutative property for the operation $+_5$ on $\mathbb{Z}_5$ using table corresponding to addition modulo 5.

47. (a) If $\vec{a}=\hat{i}+\hat{j}-\hat{k}$, $\vec{b}=2\hat{i}+3\hat{j}$ and $\vec{c}=\hat{j}-\hat{k}$, verify that $\vec{a}\times\left(\vec{b}\times\vec{c}\right)=\left(\vec{a}\cdot\vec{c}\right)\vec{b}-\left(\vec{a}\cdot\vec{b}\right)\vec{c}$

(OR)

(b) Evaluate : $\int_0^{\frac{\pi}{2}}\frac{dx}{4\sin^2 x+5\cos^2 x}$.

HIGHER SECONDARY SECOND YEAR

MATHEMATICS

MODEL QUESTION PAPER – 5

Time Allowed: 15 Min + 3.00 Hours] **[Maximum Marks:90**

Instructions: (a) Check the question paper for fairness of printing. If there is any lack of fairness, inform the Hall Supervisor immediately.

(b) Use **Blue** or **Black** ink to write and underline and pencil to draw diagrams.

PART – I

Note: (i) **All** questions are **compulsory.** $20\times1=20$

(ii) Choose the correct or most suitable answer from the given **four** alternatives. Write the option code and the corresponding answer.

PART – I

1. If A is a 3×3 matrix such that $|3adjA|=3$ then $|A|$ is equal to

(a) $\frac{1}{3}$ (b) $-\frac{1}{3}$ (c) $\pm\frac{1}{3}$ (d) ±3

2. If $A=\begin{bmatrix}3 & 1 & -1\\ 2 & -2 & 0\\ 1 & 2 & -1\end{bmatrix}$ and $A^{-1}=\begin{bmatrix}a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\\ a_{31} & a_{32} & a_{33}\end{bmatrix}$ then the value of a_{23} is

(a) 0 (b) –2 (c) –3 (d) –1

3. z_1, z_2 and z_3 be complex numbers such that $z_1+z_2+z_3=0$ and $|z_1|=|z_2|=|z_3|=1$ then $z_1^2+z_2^2+z_3^2$ is

(a) 3 (b) 2 (c) 1 (d) 0

4. The value of $i^{201}+i^{202}+i^{203}$ is

(a) 1 (b) i (c) $-i$ (d) –1

5. If $\frac{z-1}{z+1}$ is purely imaginary, then $|z|$ is

(a) $\frac{1}{2}$ (b) 1 (c) 2 (d) 3

6. If f and g are polynomials of degrees m and n respectively, and if $h(x)=(f\circ g)(x)$, then the degree of h is

(a) mn (b) $m+n$ (c) m^n (d) n^m

7. If $\sin^{-1}\frac{x}{5}+\operatorname{cosec}^{-1}\frac{5}{4}=\frac{\pi}{2}$, then the value of x is

(a) 4 (b) 5 (c) 2 (d) 3

8. The axis of the parabola $y^2-2y+8x-23=0$ is

(a) $y=-1$ (b) $x=-3$ (c) $x=3$ (d) $y=1$

9. The eccentricity of the hyperbola whose latus rectum is 8 and conjugate axis is equal to half the distance between the foci is

(a) $\frac{4}{3}$ (b) $\frac{4}{\sqrt{3}}$ (c) $\frac{2}{\sqrt{3}}$ (d) $\frac{3}{2}$

10. If the volume of the parallelepiped with $\vec{a}\times\vec{b},\vec{b}\times\vec{c},\vec{c}\times\vec{a}$ as coterminous edges is 8 cubic units, then the volume of the parallelepiped with $(\vec{a}\times\vec{b})\times(\vec{b}\times\vec{c})$, $(\vec{b}\times\vec{c})\times(\vec{c}\times\vec{a})$ and $(\vec{c}\times\vec{a})\times(\vec{a}\times\vec{b})$ as coterminous edges is,

(a) 8 cubic units (b) 512 cubic units (c) 64 cubic units (d) 24 cubic units

11. The angle between the lines $\frac{x-2}{3}=\frac{y+1}{-2}, z=2$ and $\frac{x-1}{1}=\frac{2y+3}{3}=\frac{z+5}{2}$ is

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

12. The slope of the curve $y^3-xy^2=4$ at the point where $y=2$ is

(a) -2 (b) $-\frac{1}{2}$ (c) $\frac{1}{4}$ (d) $\frac{1}{2}$

13. The point of inflection of the curve $y=(x-1)^3$ is

(a) $(0,0)$ (b) $(0,1)$ (c) $(1,0)$ (d) $(1,1)$

14. If $f(x,y)=e^{xy}$, then $\frac{\partial^2 f}{\partial x\partial y}$ is equal to

(a) xye^{xy} (b) $(1+xy)e^{xy}$ (c) $(1+y)e^{xy}$ (d) $(1+x)e^{xy}$

15. If $u(x,y)=e^{x^2+y^2}$, then $\frac{\partial u}{\partial x}$ is equal to

(a) $e^{x^2+y^2}$ (b) $2xu$ (c) x^2u (d) y^2u

16. The general solution of the differential equation $\log\left(\frac{dy}{dx}\right)=x+y$ is

(a) $e^x+e^y=c$ (b) $e^x+e^{-y}=c$ (c) $e^{-x}+e^y=c$ (d) $e^{-x}+e^{-y}=c$

17. The number of arbitrary constants in the general solutions of order n and $n+1$ are respectively

(a) $n-1, n$ (b) $n, n+1$ (c) $n+1, n+2$ (d) $n+1, n$

18. If the function $f(x)=\frac{1}{12}$ for $a<x<b$, represents a probability density function of a continuous random variable X, then which of the following cannot be the value of a and b

(a) 0 and 12 (b) 5 and 17 (c) 7 and 19 (d) 16 and 24

19. On a multiple-choice exam with 3 possible destructives for each of the 5 questions, the probability that a student will get 4 or more correct answers just by guessing is

(a) $\frac{11}{243}$ (b) $\frac{3}{8}$ (c) $\frac{1}{243}$ (d) $\frac{5}{243}$

20. In the last column of the truth table for $\neg(p \vee \neg q)$ the number of final outcomes of the truth value $'F'$ are

(a) 1 (b) 2 (c)) 3 (d) 4

PART – II

Note: (i) Answer any **SEVEN** questions. $7\times2=14$

(ii) Question number **30 is compulsory.**

21. Find the rank of the matrix : $\begin{bmatrix} 1 & -2 & -1 & 0 \\ 3 & -6 & -3 & 1 \end{bmatrix}$

22. If $z_1 = 2-i$ and $z_2 = -4+3i$, find the inverse of $\frac{z_1}{z_2}$.

23. State the reason for $\cos^{-1}\left[\cos\left(-\frac{\pi}{6}\right)\right] \neq \frac{\pi}{6}$.

24. Find the length of the latus rectum of the hyperbola $16y^2 - 9x^2 = 144$

25. Evaluate : $\lim_{x\to0}\left(\frac{\sin x}{x^2}\right)$.

26. Find the area of the region bounded by the line $6x+5y=30$, x axis and the lines $x=-1$ and $x=3$.

27. Solve : $\frac{dy}{dx}+y=e^{-x}$

28. Find the mean and variance of X, for the probability mass functions of X given below :

$$f(x)=\begin{cases} 2(x-1) & 1<x<<2 \\ 0 & \text{otherwise} \end{cases}$$

29. Let $A=\{a+\sqrt{5}b : a,b\in\mathbb{Z}\}$. Check whether the usual multiplication is a binary operation on A.

30. Find the vector equation of the plane passing through the point $(2,2,3)$ having 3,4,3 as direction ratios of the normal to the plane

PART– III

Note: (i) Answer any **SEVEN** questions. $7\times3=21$

(ii) Question number **40 is compulsory.**

31. Solve by matrix inversion method : $5x+2y=4, 7x+3y=5$

32. Solve the equation $2x^3+11x^2-9x-18=0$

33. The maximum and minimum distances of the Earth from the Sun respectively are $152\times10^6\,km$ and $94.5\times10^6\,km$. The Sun is at one focus of the elliptical orbit. Find the distance from the Sun to the other focus.

34. If $\vec{a}=\hat{i}+2\hat{j}+3\hat{k}, \vec{b}=2\hat{i}-\hat{j}+\hat{k}, \vec{c}=3\hat{i}+2\hat{j}+\hat{k}$ and $\vec{a}\times\left(\vec{b}\times\vec{c}\right)=l\vec{a}+m\vec{b}+n\vec{c}$, find the values of l,m,n.

35. Find two positive numbers whose product is 20 and their sum is minimum.

36. Find the approximate value of $\sqrt[5]{31}$.

37. Find the area of the region bounded by $2x-y+1=0$, $y=-1, y=3$ and y- axis.

38. Find the differential equation for the function $y=2\left(x^2-1\right)+ce^{-x^2}$ where c is an arbitrary constant.

39. If $X \sim B(n,p)$ such that $4P(X=4)=P(x=2)$ and $n=6$. Find the distribution, mean and standard deviation.

40. If $x+iy=\sqrt{\dfrac{a+ib}{c+id}}$, prove that $x^2+y^2=\sqrt{\dfrac{a^2+b^2}{c^2+d^2}}$

PART – IV

Note: Answer **all** the questions. $7\times5=35$

41. (a) Test for consistency and if possible, solve the system of equations $2x-y+z=2$, $6x-3y+3z=6$, $4x-2y+2z=4$. (OR)

(b) Find the all cube roots of $\sqrt{3}+i$

42. (a) Find a polynomial equation of minimum degree with rational coefficients, having $\sqrt{5}-\sqrt{3}$ as a root. (OR)

(b) If D is the midpoint of the side BC of a triangle ABC, then show by vector method that $\left|\overrightarrow{AB}\right|^2+\left|\overrightarrow{AC}\right|^2=2\left(\left|\overrightarrow{AD}\right|^2+\left|\overrightarrow{BD}\right|^2\right)$.

43. (a) Show that the straight lines $\vec{r}=\left(5\hat{i}+7\hat{j}-3\hat{k}\right)+s\left(4\hat{i}+4\hat{j}-5\hat{k}\right)$ and $\vec{r}=\left(8\hat{i}+4\hat{j}+5\hat{k}\right)+t\left(7\hat{i}+\hat{j}+3\hat{k}\right)$ are coplanar. Find the vector equation of the plane in which they lie. (OR)

(b) The volume of a cylinder equals V cubic cm, where V is a constant. Find the condition that minimize the total surface area of the cylinder.

44. (a) If the curves $ax^2+by^2=1$ and $cx^2+dy^2=1$ intersect each other orthogonally then, $\frac{1}{a}-\frac{1}{b}=\frac{1}{c}-\frac{1}{d}$. (OR)

(b) Let $z(x,y)=xe^y+ye^{-x}, x=e^{-t}, y=st^2, s,t\in\mathbb{R}$. Find $\frac{\partial z}{\partial s}$ and $\frac{\partial z}{\partial t}$.

45. (a) Prove that $\int_0^{\frac{\pi}{4}}\log(1+\tan x)dx=\frac{\pi}{8}\log 2.$ (OR)

(b) Solve : $\frac{dy}{dx}=\frac{x-y+5}{2(x-y)+7}$.

46. (a) The sum of mean and variance of a binomial distribution for five trails is 1.8. Find the distribution. (OR)

(b) Establish the equivalence property $p\leftrightarrow q\equiv(p\to q)\wedge(q\to p)$

47. (a) Solve $\cos\left(\tan^{-1}x\right)=\sin\left(\cot^{-1}\frac{3}{4}\right)$

(OR)

(b) Find the equation of the circle through the points $(1,0)$, $(-1,0)$, and $(0,1)$.

HIGHER SECONDARY SECOND YEAR

MATHEMATICS

MODEL QUESTION PAPER – 6

Time Allowed: 15 Min + 3.00 Hours] **[Maximum Marks:90**

Instructions: (a) Check the question paper for fairness of printing. If there is any lack of fairness, inform the Hall Supervisor immediately.

(b) Use **Blue** or **Black** ink to write and underline and pencil to draw diagrams.

PART – I

Note: (i) **All** questions are **compulsory.** $20\times1=20$

(ii) Choose the correct or most suitable answer from the given **four** alternatives. Write the option code and the corresponding answer.

1. If $\rho(A)=\rho([A\,|\,B])$, then the system of linear equations $AX=B$ is

(a) consistent and has a unique solution (b) consistent

(c) consistent and has infinitely many solution (d) inconsistent

2. Let A be a non-singular matrix then which one of the following is false

(a) $(\text{adj}A)^{-1}=\dfrac{A}{|A|}$ (b) I is an orthogonal matrix

(c) $adj(adjA)=|A|^n A$ (d) If A is symmetric then $\text{adj}A$ is symmetric

3. If $z=\dfrac{(\sqrt{3}+i)^3(3i+4)^2}{(8+6i)^2}$, then $|z|$ is

(a) 0 (b) 1 (c) 2 (d) 3

4. The continued product of the four values of $\left(\cos\dfrac{\pi}{3}+i\sin\dfrac{\pi}{3}\right)^{\frac{3}{4}}$ is

(a) 1 (b) -1 (c) 2 (d) -2

5. The value of $\sin^{-1}(2\cos^2 x-1)+\cos^{-1}(1-2\sin^2 x)$ is

(a) $\dfrac{\pi}{2}$ (b) $\dfrac{\pi}{3}$ (c) $\dfrac{\pi}{4}$ (d) $\dfrac{\pi}{6}$

6. The area of quadrilateral formed with foci of the hyperbolas $\dfrac{x^2}{a^2}-\dfrac{y^2}{b^2}=1$ and $\dfrac{x^2}{a^2}-\dfrac{y^2}{b^2}=-1$ is

(a) $4(a^2+b^2)$ (b) $2(a^2+b^2)$ (c) a^2+b^2 (d) $\frac{1}{2}(a^2+b^2)$

7. An ellipse has OB as semi minor axes, F and F' its foci and the angle FBF' is a right angle. Then the eccentricity of the ellipse is

(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{2}$ (c) $\frac{1}{4}$ (d) $\frac{1}{\sqrt{3}}$

8. If $\vec{a},\vec{b},\vec{c}$ are three non-coplanar unit vectors such that $\vec{a}\times(\vec{b}\times\vec{c})=\frac{(\vec{b}+\vec{c})}{\sqrt{2}}$, then the angle between $\vec{a}$ and $\vec{b}$ is

(a) $\frac{\pi}{2}$ (b) $\frac{3\pi}{4}$ (c) $\frac{\pi}{4}$ (d) π

9. The equation of the plane passing through (3,4,5) and parallel to the plane $x+2y-2z-9=0$ is

(a) $x+2y-2z=4$ (b) $x+2y-2z=3$ (c) $x+2y-2z=1$ (d) $x+2y-2z=5$

10. If $[\vec{a},\vec{b},\vec{c}]=1$, then the value of $\frac{\vec{a}\cdot(\vec{b}\times\vec{c})}{(\vec{c}\times\vec{a})\cdot\vec{b}}+\frac{\vec{b}\cdot(\vec{c}\times\vec{a})}{(\vec{a}\times\vec{b})\cdot\vec{c}}+\frac{\vec{c}\cdot(\vec{a}\times\vec{b})}{(\vec{c}\times\vec{b})\cdot\vec{a}}$ is

(a) 1 (b) -1 (c) 2 (d) 3

11. The slope of the line normal to the curve $f(x)=2\cos 4x$ at $x=\frac{\pi}{12}$ is

(i) $-4\sqrt{3}$ (ii) -4 (iii) $\frac{\sqrt{3}}{12}$ (iv) $4\sqrt{3}$

12. The number given by the Rolle's theorem for the function $x^3-3x^2, x\in[0,3]$ is

(a) 1 (b) $\sqrt{2}$ (c) $\frac{3}{2}$ (d) 2

13. If $f(x,y,z)=xy+yz+zx$, then f_x-f_z is equal to

(a) $z-x$ (b) $y-z$ (c) $x-z$ (d) $y-x$

14. If $f(x)=\int_0^x t\cos t\,dt$, then $\frac{df}{dx}=$

(a) $\cos x-x\sin x$ (b) $\sin x+x\cos x$ (c) $x\cos x$ (d) $x\sin x$

15. The value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\frac{\sin x}{2+\cos x}dx$ is

(a) 0 (b) 2 (c) $\log 2$ (d) $\log 4$

16. The order and degree of the differential equation $\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^{1/3}+x^{1/4}=0$ are respectively.

(a) 2, 3 (b) 3, 3 (c) 2, 6 (d) 2, 4

17. The solution of the differential equation $2x\frac{dy}{dx}-y=3$ represents

(a) straight lines (b) circles (c) parabola (d) ellipse

18. A random variable X has binomial distribution with $n=25$ and $p=0.8$ then standard deviation of X is

(a) 6 (b) 4 (c) 3 (d) 2

19. If $f(x)=\begin{cases}2x & 0\le x\le a\\ 0 & \text{otherwise}\end{cases}$ is a probability density function of a random variable, the value then of a is

(a) 1 (b) 2 (c) 3 (d) 4

20. Subtraction is not a binary operation on

(a) $\mathbb{R}$ (b) $\mathbb{Z}$ (c) $\mathbb{N}$ (d) $\mathbb{Q}$

PART – II

$7\times2=14$

Note: (i) Answer any **SEVEN** questions.

(ii) Question number **30 is compulsory.**

21. If $adj(A)=\begin{bmatrix}0 & -2 & 0\\ 6 & 2 & -6\\ -3 & 0 & 6\end{bmatrix}$, find A^{-1}.

22. Express $-1+i\sqrt{3}$ in polar form.

23. Find centre and radius of the circle $2x^2+2y^2-6x+4y+2=0$.

24. Find the angle between the straight line $\vec{r}=\left(2\hat{i}+3\hat{j}+\hat{k}\right)+t\left(\hat{i}-\hat{j}+\hat{k}\right)$ and the plane

$2x-y+z=5$.

25. Explain why Lagrange mean value theorem is not applicable to the function

$f(x)=\left|\frac{1}{x}\right|,\ x\in[-1,1]$.

26. Evaluate : $\int_0^1 \frac{|x|}{x}\,dx$

27. Form the differential equation of the family of parabolas $y^2=4ax$, where a is an arbitrary constant.

28. Compute $P(X=k)$ for the binomial distribution, $B(n,p)$ where $n=10,\ p=\frac{1}{5},\ k=4$

29. Write the statements in words corresponding to $\neg p$, $q \vee \neg p$, where p is 'It is cold' and q is 'It is raining.'.

30. Find the value of $\cos^{-1}\left(\frac{1}{2}\right)-2\sin^{-1}\left(-\frac{1}{2}\right)$

PART– III

Note: (i) Answer any **SEVEN** questions. $7\times3=21$

(ii) Question number **40 is compulsory.**

31. Find the adjoint of the matrix $A=\begin{bmatrix}1 & 3\\ 2 & -5\end{bmatrix}$ and verify that $A(adjA)=(adjA)A=|A|I$.

32. Show that the points 1, $-\frac{1}{2}+\frac{i\sqrt{3}}{2}$ and $-\frac{1}{2}-\frac{i\sqrt{3}}{2}$ are the vertices of the equilateral triangle.

33. Find the equation of the hyperbola with vertices $(0,\pm4)$ and foci $(0,\pm6)$.

34. If the two lines $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-1}{4}$ and $\frac{x-3}{1}=\frac{y-m}{2}=z$ intersect at a point, find the value of m.

35. If $w(x,y)=xy+\sin(xy)$, then prove that $\frac{\partial^2 w}{\partial y\partial x}=\frac{\partial^2 w}{\partial x\partial y}$

36. Evaluate : $\int_0^{\frac{\pi}{3}}\frac{\sec x\tan x}{1+\sec^2 x}dx$.

37. Solve : $\frac{dy}{dx}=\sqrt{\frac{1-y^2}{1-x^2}}$.

38. Form the differential equation of $y=e^{3x}(C\cos 2x+D\sin 2x)$, where C and D are arbitrary constants.

39. Solve : $y^2+x^2\frac{dy}{dx}=xy\frac{dy}{dx}$

40. Find the equation of the tangent to the curve $x^2y-x=y^3-8$ at $x=0$

PART – IV

Note: Answer **all** the questions. $7\times5=35$

41. (a) Find the inverse of the non-singular matrix $A=\begin{bmatrix}2 & 1 & 1\\ 3 & 2 & 1\\ 2 & 1 & 2\end{bmatrix}$, by elimentary transformations. (OR)

(b) If $z=x+iy$ and $\arg\left(\frac{z-1}{z+1}\right)=\frac{\pi}{2}$, then show that $x^2+y^2=1$

42. (a) Solve the equation $6x^4-5x^3-38x^2-5x+6=0$ if it is known that $\frac{1}{3}$ is a solution.(OR)

(b) Solve $\tan^{-1}2x+\tan^{-1}3x=\frac{\pi}{4}$, if $6x^2<1$.

43. (a) An elliptical whispering room has height 5m and width 26m.Where should two persons stand if they would like to whisper back and forth and be heard. (OR)

(b) Show that the four points $(6,-7,0),(16,-19,-4),(0,3,-6),(2,-5,10)$ lie on a same plane.

44. (a) Show that the straight lines $x+1=2y=-12z$ and $x=y+2=6z-6$ are skew and hence find the shortest distance between them. (OR)

(b) If we blow air into a balloon of spherical shape at a rate of $1000cm^3$ per second. At what rate the radius of the baloon changes when the radius is 7cm? Also compute the rate at which the surface area changes.

45. (a) If an initial amount A_0 of money is invested at an interest rate r compounded n times a year, the value of the investment after t years is $A=A_0\left(1+\frac{r}{n}\right)^{nt}$. If the interest is compounded continuously, (that is $n\to\infty$) show that the amount after t years is $A=A_0e^{rt}$. (OR)

(b) If $u=\sec^{-1}\left(\frac{x^3-y^3}{x+y}\right)$, show that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=2\cot u$

46. (a) The curve $y=(x-2)^2+1$ has a minimum point at P. A point Q on the curve is such that the slope of PQ is 2. Find the area bounded by the curve and the chord PQ. (OR)

(b) Prove that $p\to(\neg q\vee r)\equiv\neg p\vee(\neg q\vee r)$ using truth table.

47. (a) An equation relating to the stability of an aircraft is given by $\frac{dv}{dt}=g\cos\alpha-kv$, where g , α , k are constants and v is the velocity. Obtain an expression in terms of v if $v=0$ when $t=0$. (OR)

(b) Four fair coins are tossed once. Find the probability mass function, mean and variance for number of heads occurred.